A Game-Theoretic Approach to Relay Selection in Cooperative Wireless Networks

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***Abstract*—In this paper, distributed relay selection in cooperative wireless networks is modeled as a Chinese restaurant game (CRG). Specifically, the CRG is used to model strategic relay selection decisions of source nodes, taking into account negative network externality due to the potential sharing of relay nodes among source nodes. In turn, a distributed relay selection algorithm is proposed and shown to converge to a Nash Equilibrium grouping. Simulation results verify the efficiency of the proposed distributed algorithm when compared with other relay selection schemes, and demonstrate that it yields a network sum-rate that is comparable with that of centralized relay selection.**

Keywords*—*Amplify-and-forward, centralized, cooperation, distributed, game-theory, Nash Equilibrium, relay selection

I. Introduction

Relay selection in cooperative wireless networks has emerged as an effective technique to achieve full diversity gains while maintaining spectral and energy efficiency, and minimizing complexity and overhead [1] [2] [3]. Specifically, relay selection aims at selecting the best/optimal relay(s) from a set of potential relays to cooperate with source nodes, and form multi-hop communication links, for coverage extension and improved network connectivity [4].

Recently, game-theoretic approaches have received much attention in the design of adaptive and distributed relay selection schemes for wireless networks. Different game models have been studied and applied in the modeling of the complex interactions of source and relay nodes in the relay selection process. For instance, in [5], the authors propose the use of a two-level Stackelberg Game for distributed relay selection and power allocation. Additionally, the authors prove that the game converges to a unique optimal equilibrium and gives a comparable performance to centralized schemes. A Stackelberg game in extensive-form has been proposed in [6] for power allocation for bidirectional cooperation. Specifically, the authors show that Nash Equilibrium is closely attainable and fair power allocation is achieved. In [7], a game-theoretical model for relay selection in randomized orthogonal space-time coding links is proposed to minimize energy waste and overhead for node management. In [8], distributed relay selection has been proposed and modeled as a non-cooperative, mixed strategy repeated game. Moreover, an adaptive learning algorithm has been incorporated into the game model to allow network nodes to learn optimal strategies of relay selection in dynamic environments.

Naturally, source nodes aim at selecting the relay nodes with good channel conditions and high transmit power to maximize their utilities. However, due to the limited transmission resources at each relay node, the more source nodes share a relay, the less utility each source node may achieve when that relay simultaneously serves multiple source nodes (i.e. negative network externality). Thus, the Chinese restaurant game (CRG)—derived from the Chinese restaurant process [9]—has been proposed as an effective means to address the problem of negative network externality [10][11]. Generally speaking, the CRG models the strategic behavior of each customer entering a Chinese restaurant and deciding whether to select a free table or share a table with other customers. The strategic decision of each rational customer is based on the sizes of available tables and the number of customers sharing each table.

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In this paper, the relay selection process in cooperative wireless networks with multiple source and relay nodes is modeled as a sequential Chinese restaurant game, in which each rational source node sequentially selects the best relay—in terms of achievable rate—from a set of available relay nodes. The main contributions can be summarized as follows.

- A distributed iterative relay selection algorithm is proposed and shown to converge to a Nash Equilibrium grouping.
- Compared the proposed distributed algorithm with other relay selection schemes to verify its efficiency, and also with centralized relay selection to illustrate its low complexity, and comparable achievable network sum-rate.

In the remainder of this paper, the system model is presented in Section II. In Section III, relay selection is a modeled as a Chinese restaurant game and its properties are discussed. The distributed relay selection algorithm is presented in Section IV while the centralized relay selection problem is formulated in Section V. Simulation results are presented in Section VI while the conclusions are drawn in Section VII.

II. System Model

Consider an ad-hoc wireless network consisting of N source nodes ($N \geq 2$), denoted $S_1, S_2, \ldots, S_N$. Each source node S_j for $j \in \mathcal{N}$, where $\mathcal{N} = \{1, 2, \ldots, N\}$, is assumed to have its own data symbol x_j and aims at communicating it to a common destination node D via a relay node R_k. In particular, there are K amplify-and-forward relay nodes, each with transmit power of P_{R_k}, for $k \in \mathcal{K}$, where $\mathcal{K} = \{1, 2, \ldots, K\}$. The channel between any two nodes is modeled as a narrowband Rayleigh channel with additive white Gaussian noise (AWGN). Specifically, let $h_{j,k}$ be the channel coefficient representing the channel between any two nodes j and k, such thats $h_{j,k} \sim \mathcal{CN}(0, \sigma^2_{j,k})$, where $\sigma^2_{j,k} = d^{-\nu}_{j,k}$ is the channel gain, with $d_{j,k}$ being the inter-node distance and ν is the path-loss exponent. At each source/relay node, perfect channel estimation is assumed. Additionally, it is assumed that there is no direct link between the source nodes and the destination node.

Communication between the source nodes and the destination node is performed in a TDMA fashion over a total of $N+1$ time-slots and is split into two phases, namely the broadcasting phase (of N time-slots) and the cooperation phase (of one time-slot).

A. Broadcasting Phase

In this phase, each source node S_j for $j \in \mathcal{N}$ is assigned a time-slot T_j in which it broadcasts its data symbol x_j, which is received by each relay node R_k. Specifically, the received signal $y_{j,k}$ at relay node R_k for $k \in \mathcal{K}$ is written as

$$y_{j,k} = \sqrt{P_{B_j}} h_{j,k} x_j + \eta_{j,k}, \tag{1}$$

$$\mathcal{R}_j(P_{C_{j,k}}) = \frac{1}{N+1}\log_2\left(1+\sum_{k=1}^{K}\mathcal{I}_{j,k}\frac{P_{B_j}P_{C_{j,k}}|h_{j,k}|^2|h_{k,d}|^2}{N_0\varrho_N(P_{B_j}|h_{j,k}|^2+P_{C_{j,k}}|h_{k,d}|^2+N_0)}\right). \quad (11)$$

$$\mathcal{R}_j\left(P_{R_k}, n_{R_k}\right) = \frac{1}{N+1}\log_2\left(1+\sum_{k=1}^{K}\mathcal{I}_{j,k}\frac{P_{B_j}\left(\frac{P_{R_k}}{n_{R_k}}\right)|h_{j,k}|^2|h_{k,d}|^2}{N_0\varrho_N(P_{B_j}|h_{j,k}|^2+\left(\frac{P_{R_k}}{n_{R_k}}\right)|h_{k,d}|^2+N_0)}\right). \quad (12)$$

where P_{B_j} is the broadcast transmit power of source node S_j, and $\eta_{j,k}$ is the zero-mean N_0-variance complex AWGN sample at relay R_k. At the end of the broadcasting phase, each relay R_k will have received a set of N signals $\{y_{j,k}\}_{j=1}^{N}$.

B. Cooperation Phase

In the cooperation phase, it is assumed that each source node is served by only one relay; however, a single relay can serve multiple source nodes (as discussed in Section III). Let $\mathcal{I}_k$ be the set of source nodes that select relay R_k for their data transmission, where $0 \leq |\mathcal{I}_k| \leq N$. Also, define $\mathcal{I}_{j,k}$ as

$$\mathcal{I}_{j,k} = \begin{cases} 1, & \text{if source } S_j \text{ selects relay } R_k \\ 0, & \text{otherwise} \end{cases}. \quad (2)$$

For multiuser detection at the destination, each source node S_i is assigned a signature waveform $c_i(t)$, such that $\rho_{i,j} \triangleq (1/T_s)\int_0^{T_s} c_i(t)c_j^*(t)dt$ for $i \neq j$, with $\rho_{i,i} = 1$, and T_s being the symbol duration. Each selected relay R_k forms a signal $\mathcal{X}_k(t)$, as given by

$$\mathcal{X}_k(t) = \sum_{i=1}^{N}\mathcal{I}_{i,k}\sqrt{P_{C_{i,k}}}\beta_{i,k}y_{i,k}c_i(t), \quad (3)$$

where $P_{C_{i,k}}$ is the cooperative power assigned by relay R_k to source S_i, and $\beta_{i,k} = \sqrt{\frac{1}{P_{B_i}|h_{i,k}|^2+N_0}}$ is a normalization factor. Assuming perfect timing synchronization, all selected relays simultaneously transmit their signals, which are received at the destination as

$$y_d(t) = \sum_{k=1}^{K} h_{k,d}\mathcal{X}_k(t) + \eta_d(t), \quad (4)$$

where $\eta_d(t)$ is the AWGN process at the destination. Substituting (1) and (3) into (4) yields

$$y_d(t) = \sum_{k=1}^{K}\sum_{i=1}^{N}\mathcal{I}_{i,k}\beta_{i,k}\sqrt{P_{B_i}P_{C_{i,k}}}h_{i,k}h_{k,d}x_ic_i(t) + \bar{\eta}_d(t), \quad (5)$$

where $\bar{\eta}_d(t)$ is the equivalent noise term, given by

$$\bar{\eta}_d(t) = \eta_d(t) + \sum_{k=1}^{K}\sum_{i=1}^{N}\mathcal{I}_{i,k}\sqrt{P_{C_{i,k}}}\beta_{i,k}h_{k,d}\eta_{i,k}c_i(t). \quad (6)$$

At the destination, multiuser detection is performed on $y_d(t)$ to extract each symbol x_j for $j \in \mathcal{N}$, as

$$y_{j,d} = \sum_{k=1}^{K}\sum_{i=1}^{N}\mathcal{I}_{i,k}\beta_{i,k}\sqrt{P_{B_i}P_{C_{i,k}}}h_{i,k}h_{k,d}x_i\rho_{i,j} + \bar{\eta}_{j,d}. \quad (7)$$

In this work, it is assumed that $\rho_{i,j} = \rho, \forall i \neq j$. Therefore, the decorrelated received signal $\tilde{y}_{j,d}$ is obtained as

$$\tilde{y}_{j,d} = \sum_{k=1}^{K}\mathcal{I}_{j,k}\beta_{j,k}\sqrt{P_{B_j}P_{C_{j,k}}}h_{j,k}h_{k,d}x_j + \tilde{\eta}_{j,d}, \quad (8)$$

where $\tilde{\eta}_{j,d} \sim \mathcal{CN}(0, N_0\varrho_N(\sum_{k=1}^{K}\mathcal{I}_{j,k}P_{C_{j,k}}\beta_{j,k}^2|h_{k,d}|^2+1))$, and

$$\varrho_N = \frac{1+(N-2)\rho}{1+(N-2)\rho-(N-1)\rho^2}. \quad (9)$$

Using maximum-ratio-combining, the instantaneous SNR of the received decorrelated signal is expressed as

$$\gamma_j = \sum_{k=1}^{K}\mathcal{I}_{j,k}\frac{P_{B_j}P_{C_{j,k}}|h_{j,k}|^2|h_{k,d}|^2}{N_0\varrho_N(P_{B_j}|h_{j,k}|^2+P_{C_{j,k}}|h_{k,d}|^2+N_0)}. \quad (10)$$

Hence, the achievable rate of source S_j as a function of the allocated cooperative power $P_{C_{j,k}}$ is given by (11). As mentioned earlier, only one relay is selected by source S_j for cooperative transmission (i.e. $\sum_{k=1}^{K}\mathcal{I}_{j,k} = 1$). Moreover, there is a transmit power constraint per source/relay node (i.e. $P_{B_j} \leq P_B$, $\forall j \in \mathcal{N}$ and $P_{R_k} \leq P_R$, $\forall k \in \mathcal{K}$). Further, $\sum_{j=1}^{N}\mathcal{I}_{j,k}P_{C_{j,k}} \leq P_{R_k}, \forall k \in \mathcal{K}$. Also, it is assumed that each source node perfectly knows the available transmit power at each relay node.

III. Relay Selection as a Chinese Restaurant Game

The Chinese restaurant game model assumes a restaurant with N customers and K tables each with a specific size [11]. By analogy, the N customers are represented by the N source nodes and the K tables are modeled by K relays. Additionally, the sizes of the tables are modeled by the available transmit relay power P_{R_k} at each relay R_k, for $k \in \mathcal{K}$. Notice that $\mathcal{K}$ represent the action set of each source node such that $r_j \in \mathcal{K}$ is source S_j's relay selection. Now, let n_{R_k} be the number of source nodes selecting relay R_k. Thus, at each relay R_k, the power allocated $P_{C_{j,k}}$ to each source S_j is a function of the number of source nodes n_{R_k} sharing that relay (i.e. $P_{C_{j,k}} = \frac{1}{n_{R_k}}P_{R_k}, \forall j \in \mathcal{I}_k$). Thus, the achievable rate in (11) can be re-written as in (12). In turn, the achievable rate $\mathcal{R}_j$ of source S_j is an increasing function of P_{R_k}, but a decreasing function of n_{R_k} (i.e. negative network externality). Now, let $\mathbf{n_{-j}} = \{n_{-j,R_1}, n_{-j,R_2}, \ldots, n_{-j,R_K}\}$, where n_{-j,R_k} denotes the number of source nodes choosing relay R_k, except for source node S_j. Since the source nodes are rational, they aim at maximizing their achievable rates. Also, a source node's relay selection is affected by the other source nodes' relay selections. Thus, a rational source node S_j chooses a relay according to

$$BE_j\left(\mathbf{n_{-j}}\right) = \arg\max_{k\in\mathcal{K}}\ \mathcal{R}_j(P_{R_k}, n_{-j,R_k}+1), \quad (13)$$

which is the best response action of source node S_j. Based on (12) and (13), it should be noted that each relay's transmit power is distributed equally over the source nodes sharing it. Therefore, each source node's best response is to select the rate-maximizing relay, given the relay selections of the other source nodes.

Now, in order to establish Nash Equilibrium, a few definitions must be stated [10].

Definition 1 (Equilibrium Grouping): An equilibrium source node grouping $\mathbf{n}^*$ is defined as $\mathbf{n}^* = \{n_{R_1}^*, \ldots, n_{R_K}^*\}$, where n_k^* (for $k \in \mathcal{K}$) is the number of source nodes selecting relay node R_k, at the end of the relay selection game.

Definition 2 (Relay Selection Profile): Let $\mathbf{r}^* = \{r_1^*, r_2^*, \ldots, r_N^*\}$ be the equilibrium relay selection profile, where r_j^* is the selected relay by source S_j, such that $r_j^* \in \mathcal{K}, \forall j \in \mathcal{N}$. Thus, the best response relay selection by source S_j can equivalently be written as

$$BE_j\left(\mathbf{r}^*_{-\mathbf{j}}\right) = r_j^*, \tag{14}$$

where $\mathbf{r}^*_{-\mathbf{j}} = \{r_1^*, \ldots, r_{j-1}^*, r_{j+1}^*, \ldots, r_N^*\}$, $\forall j \in \mathcal{N}$.

Definition 3 (Nash Equilibrium Grouping): A Nash Equilibrium grouping $\mathcal{G}\left(\mathbf{r}^*, \mathbf{n}^*\right)$ is a relay selection profile $\mathbf{r}^*$ and associated source nodes grouping $\mathbf{n}^*$, where each source node uses its best response to select the rate-maximizing relay such that none of the source nodes have an incentive to deviate from their relay selections.

In [10], the necessary and sufficient conditions of Nash Equilibrium grouping in the Chinese restaurant game have been shown by the following Theorem.

Theorem 1 (Nash Equilibrium): Given the set of source nodes S_j for $j \in \mathcal{N}$ and the set of relay nodes R_k for $k \in \mathcal{K}$, any Nash Equilibrium grouping $\mathcal{G}\left(\mathbf{r}^*, \mathbf{n}^*\right)$ in the relay selection game should satisfy

$$\mathcal{R}_j(P_{R_k}, n^*_{R_k}) \geq \mathcal{R}_j(P_{R_l}, n^*_{-j,R_l} + 1), \quad \forall j \in \mathcal{N}, \tag{15}$$

provided $n^*_{R_k}, n^*_{-j,R_l} > 0$, $\forall k, l \in \mathcal{K}$. Notice that although the source nodes may have different utility functions (i.e. achievable rates), the original proof of Theorem 1 in [10] is still tenable in our relay selection game. Additionally, in [10], it has been established that there exists at least one Nash Equilibrium in the Chinese restaurant game. This is clear, since a pair of source nodes may exchange their relay selections in one Nash Equilibrium to obtain another one without violating (15), and thus leading to the following corollary.

Corollary 1 (Existence of Nash Equilibrium): There exists at least one Nash Equilibrium grouping in our relay selection game.

It is noteworthy that in our relay selection game, the achievable rate function of each source node is strictly convex, and a continuous random variable, with the probability that the achievable rate of a source node resulting from two different relays being zero. In turn, the following definition can be stated.

Definition 4 (Strictly-Dominated Relay Selection): Given a relay selection $\mathbf{r}$ and source nodes grouping $\mathbf{n}$, a relay selection $r'_j \in \mathcal{K}$ of source node S_j (for $j \in \mathcal{N}$) is strictly dominated by relay selection $r_j \in \mathcal{K}$, if for relay selections $\mathbf{r}_{-\mathbf{j}} = \{r_1, \ldots, r_{j-1}, r_{j+1}, \ldots, r_N\}$ by the other source nodes, $\mathcal{R}_j(P_{R_{r_j}}, n_{R_{r_j}}) > \mathcal{R}_j(P_{R_{r'_j}}, n_{-j,R_{r'_j}} + 1)$, provided $n_{R_{r_j}}, n_{-j,R_{r'_j}} > 0$, $\forall r_j, r'_j \in \mathcal{K}$.

This definition simply states that for a specific $\mathbf{r}$ and $\mathbf{n}$ pair, $r'_j \in \mathcal{K}$ is not a best-response relay selection to the relay selection $\mathbf{r}_{-\mathbf{j}}$ by the other source nodes, since there is a relay r_j that yields a strictly higher achievable rate than r'_j for source node S_j. This leads to the following corollary.

Corollary 2 (Strict Nash Equilibrium): Any Nash Equilibrium grouping in the relay selection game is strict.

Based on Corollaries 1 and 2, although there can be several Nash equilibria, each one of them is strict.

IV. Distributed Relay Selection Algorithm

A. Algorithm Description

Although the original problem is formulated such that source nodes make their relay selection decisions simultaneously, it would be impractical to submit relay selections simultaneously. Instead, a sequential relay selection algorithm is proposed, based on two stages. In Stage 1, source S_j is the j^{th} source node to select a relay (i.e. in a predetermined sequential order). Since $\mathbf{n}_{-\mathbf{j}} = \{n_{-j,R_1}, n_{-j,R_2}, \ldots, n_{-j,R_K}\}$ then $\sum_{k=1}^{K} n_{-j,R_k} = j-1$ is the number of relay selections observed by source S_j. Note that $\mathbf{n}_{-\mathbf{j}}$ may not be fully observable by source S_j since source nodes $S_{j+1}, \ldots, S_N$ make their relay selection decisions after source S_j. In such case, each source S_j selects a relay that maximizes its achievable rate based on what has been observed (i.e. relay selections of previous source nodes), and in accordance with (13). It should be noted that Stage 1 in the proposed algorithm is an adaptation from the equilibrium grouping finding algorithm proposed in [10]. Nevertheless, the original algorithm does not work in our model since it relies on the assumption that all players share the same utility function. Consequently, the output grouping from the algorithm in [10] will not necessarily be a Nash equilibrium grouping in our relay selection game. To resolve this issue, Stage 2 is proposed, where a source node sharing a relay with other source nodes may deviate to select a relay that yields a better achievable rate. Therefore, Stage 2 involves a series of deviations in which some source nodes will only deviate to strictly improve their achievable rates by making best-response relay selections. This in turn may trigger other source nodes to deviate and select different rate-improving relays. This process repeats until no source node has an incentive to deviate, as no other relays can strictly improve their achievable rates, leading to a Nash Equilibrium grouping $\mathcal{G}\left(\mathbf{r}^*, \mathbf{n}^*\right)$. Table 1 outlines the proposed distributed relay selection algorithm, where it is noteworthy that given a Nash Equilibrium grouping, the achievable rate of each source node can be determined.

Algorithm 1 : Distributed Relay Selection

Stage 1: (Relay Selection)

FOR $j = 1 : N$
  $\mathcal{R}_j = 0$, $r_j = 0$;
  FOR $k = 1 : K$
    IF $\mathcal{R}_j(P_{R_k}, n_{-j,R_k} + 1) > \mathcal{R}_j$
      $r_j = k$, and $\mathcal{R}_j = \mathcal{R}_j(P_{R_k}, n_{-j,R_k} + 1)$;
    END IF
  END FOR
  $n_{R_{r_j}} = n_{-j,R_{r_j}} + 1$;
  Update $\mathbf{r} = \{r_1, \ldots, r_N\}$, $\mathbf{n} = \{n_{R_1}, \ldots, n_{R_K}\}$, and $\mathcal{R}_i$ for $i \in \{1, 2, \ldots, N\}$ and $i \neq j$;
END FOR

Stage 2: (Deviations)

Initialize BestResponse $\leftarrow$ TRUE;
WHILE (BestResponse)
  BestResponse $\leftarrow$ FALSE;
  FOR $j = 1 : N$
    FOR $k = 1 : K, k \neq r_j$
      IF $\mathcal{R}_j(P_{R_k}, n_{-j,R_k} + 1) > \mathcal{R}_j$
        $r_j = k$, and $\mathcal{R}_j = \mathcal{R}_j(P_{R_k}, n_{-j,R_k} + 1)$;
        Update $\mathbf{r} = \{r_1, \ldots, r_N\}$, $\mathbf{n} = \{n_{R_1}, \ldots, n_{R_K}\}$, and $\mathcal{R}_i$ for $i \in \{1, 2, \ldots, N\}$ and $i \neq j$;
        BestResponse $\leftarrow$ TRUE;
      END IF;
    END FOR
  END FOR
END WHILE

Output: $\mathbf{r}^* = \{r_1^*, \ldots, r_N^*\}$ and $\mathbf{n}^* = \{n^*_{R_1}, \ldots, n^*_{R_K}\}$

B. Convergence

Theorem 3 (Convergence): The proposed distributed relay selection algorithm converges to a Nash Equilibrium grouping $\mathcal{G}(\mathbf{r}^*, \mathbf{n}^*)$ in a finite number of iterations.

Proof: The key points to the convergence proof are based on the fact that a rational source node will only deviate to a relay that strictly improves its achievable rate, and that the source nodes' achievable rates cannot increase indefinitely.

Based on the proposed sequential relay selection algorithm, source nodes that are first to select a relay are more likely to deviate and change their relay selections. This specifically happens when the same relay is selected later by at least one other source node. Also, note that a source node that is not sharing a relay node with any other source node at the end of Stage 1 may not deviate in Stage 2 unless that relay was selected later by another source node or another relay becomes strictly more preferable to that source node. Moreover, when a source node deviates and selects a different relay that strictly improves its achievable rate, the source nodes sharing the previous relay also improve their achievable rates as the allocated relay power per source node increases. Now, if the newly selected relay is shared by at least one other source node, then that source node may only deviate and select another relay if it strictly improves its achievable rate. Otherwise, no further deviations will take place, leading to a Nash Equilibrium grouping $\mathcal{G}(\mathbf{r}^*, \mathbf{n}^*)$. □

V. Centralized Relay Selection

The centralized relay selection problem is formulated as a mixed integer nonlinear programming (MINLP) problem as given by

$$\max \quad \frac{1}{N+1} \times \sum_{j=1}^{N} \log_2\left(1+\sum_{k=1}^{K} \mathcal{I}_{j,k} \frac{P_{B_j} P_{C_{j,k}} |h_{j,k}|^2 |h_{k,d}|^2}{N_0 \varrho_N (P_{B_j}|h_{j,k}|^2 + P_{C_{j,k}}|h_{k,d}|^2 + N_0)}\right)$$

$$\text{s.t.} \quad \sum_{j=1}^{N} \mathcal{I}_{j,k} = n_{R_k}, \qquad \forall k \in \{1,2,\ldots,K\}, \tag{16a}$$

$$\sum_{k=1}^{K} \mathcal{I}_{j,k} = 1, \qquad \forall j \in \{1,2,\ldots,N\}, \tag{16b}$$

$$n_{R_k} \cdot P_{C_{j,k}} \leq P_{R_k},\ \forall j \in \{1,2,\ldots,N\} \text{ and } \forall k \in \{1,2,\ldots,K\}, \tag{16c}$$

$$P_{C_{j,k}} \geq 0, \qquad \forall j \in \{1,2,\ldots,N\} \text{ and } \forall k \in \{1,2,\ldots,K\}, \tag{16d}$$

$$\mathcal{I}_{j,k} \in \{0,1\}, \qquad \forall j \in \{1,2,\ldots,N\} \text{ and } \forall k \in \{1,2,\ldots,K\}, \tag{16e}$$

$$n_{R_k} \in \{0,1,\ldots,N\}, \quad \forall k \in \{1,2,\ldots,K\}. \tag{16f}$$

The first constraint defines the number of source nodes n_{R_k} sharing relay R_k; while the second constraint ensures that each source node selects only one relay. The third constraint ensures that the power constraint at each relay R_k is satisfied. The last three constraints define the range of values each decision variable can take.

Remark 1: Since the achievable rate of each source node is a strictly monotonically increasing function of the allocated relay power, then the total power constraint at each relay is always satisfied (i.e. $n_{R_k} \cdot P_{C_{j,k}} = P_{R_k}, \forall k \in \mathcal{K}$).

It is noteworthy that the formulated MINLP centralized relay selection problem is in general NP-hard and often quite difficult to solve [12][13]. Table I summarizes the total number of continuous and integer decision variables and constraints for the centralized relay selection problems as a function of the number of source and relay nodes, N and K, respectively. Due to high evaluation costs, such problems can no longer be accurately solved and centralized algorithms are hoped to compute good approximate solutions. Even so, computational complexity becomes extremely prohibitive in dense networks.

TABLE I. Summary of the Number of Decision Variables and Constraints Required for Centralized Relay Selection

Number of Decision Variables		Number of Constraints
Continuous	Integer	
$K \cdot N$	$K \cdot N + K$	$K + N + K \cdot N$

VI. Simulation Results

In this section, the proposed distributed relay selection algorithm is evaluated and compared to centralized[2] relay selection. The simulations assume signature waveforms with $\rho = 0.25$, path-loss exponent $\nu = 3$, source transmit power of 50 mW, and noise variance $N_0 = 10^{-5}$ W. The total network relay power per time-slot is set to 500 mW. The simulation results are averaged over 10000 independent runs with randomly generated channel coefficients for each run. The simulated network topology is illustrated in Fig. 1.

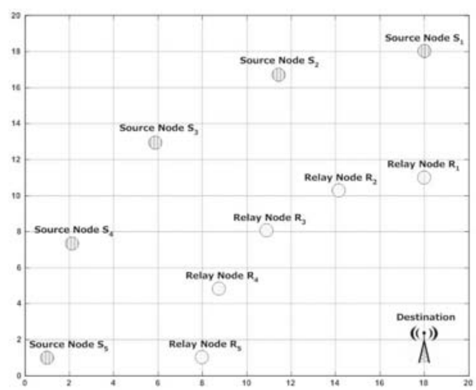

Fig. 1. Network Topology

In the simulations, the following three scenarios are evaluated:

Scenario 1: 5 Source Nodes and 5 Relay Nodes:
In Scenario 1, all 5 source and 5 relay nodes in the network are operational.

Scenario 2: 5 Source Nodes and 3 Relay Nodes:
In this Scenario, all 5 source nodes are operational but only relay nodes R_1, R_3 and R_5 are active.

Scenario 3: 3 Source Nodes and 5 Relay Nodes:
In Scenario 3, only source nodes S_1, S_3 and S_5 are operational but all 5 relay nodes are active.

In each of the above scenarios, two relay power allocation schemes are considered:

Equal Relay Power (ERP):
All relay nodes have equal power of 100 mW under Scenarios 1 and 3. As for Scenario 2, the total relay power is split equally on the 3 relay nodes[3] (i.e. each relay is allocated 166.7 mW).

Unequal Relay Power (URP):
Transmit power of relay R_1 is 300 mW under all three

[2]The centralized MINLP relay selection problem is solved using MIDACO [14] with optimization tolerance set to 0.01.

[3]This relay power allocation ensures fairness in comparing all three scenarios.

scenarios, while the transmit power of the other relays is 50 mW under Scenarios 1 and 3. In Scenario 2, the other two relays each have transmit power of 100 mW.

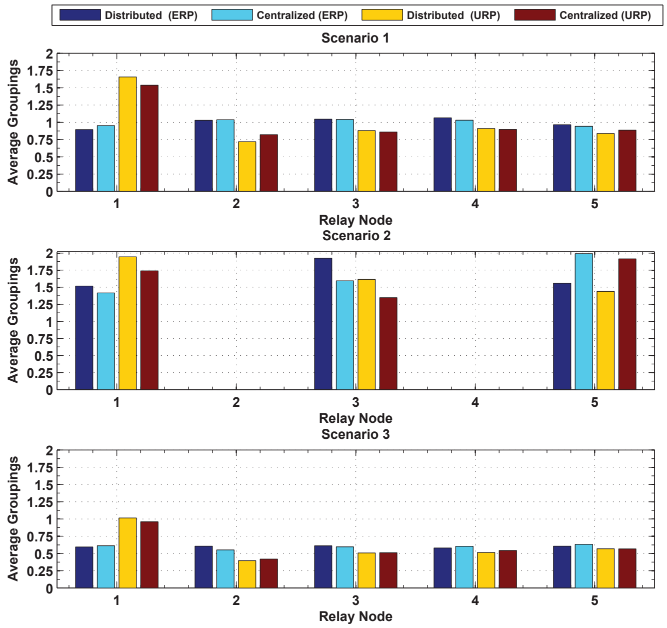

Fig. 2. Average Groupings Under the Distributed and Centralized Algorithms

In Fig. 2, the average groupings $\mathbf{n}^*$ of each relay under the different scenarios and relay power allocation schemes are illustrated under the distributed and centralized algorithms. Under Scenario 1, both the distributed and centralized algorithms lead to comparable groupings, irrespective of the relay power allocation scheme. The slight discrepancy is due to the fact that the distributed algorithm follows a sequential order in relay selection, which might not be optimal; while the centralized algorithm determines the optimal relay assignment to each source node. Additionally, it can be seen that each relay under the distributed algorithm with the ERP allocation scheme is selected on average by one source node. However, under the URP allocation scheme, relay R_1 is selected—as expected—by a higher number of source nodes than the other relays, which is due to the fact that it has higher transmit power, and thus is more favorable. This observation is seen in all three scenarios, under the distributed algorithm. Now, the average groupings under Scenario 2—where relays R_2 and R_4 are inactive—are higher than the other two scenarios. This is intuitively attributed to the fewer number of active relays, which leads to higher competition of the 5 source nodes for relays that maximize their rates. For instance, under the ERP allocation scheme, relay R_3 (R_5) is selected on average by two sources under the distributed (centralized) algorithm. Lastly, the observations of Scenario 1 can be applied to Scenario 3.

Fig. 3 shows the percentage of relay selection of the source nodes under Scenario 3 with ERP allocation. Under the distributed (centralized) algorithm, source S_1 selects relay R_1 52% (57%) of the time but selects relay R_5 less than 1% of the time. This is expected since relay R_1 is the closest to source S_1; while relay R_5 is the farthest from it. An opposite observation is made for source S_5. As for source S_3, it is closest to relay R_3 and closer to relays R_2 and R_4 than relays R_1 and R_5, which explains the corresponding relay selections. Similar observations have been made on the URP allocation scheme but not shown here due to space limitation.

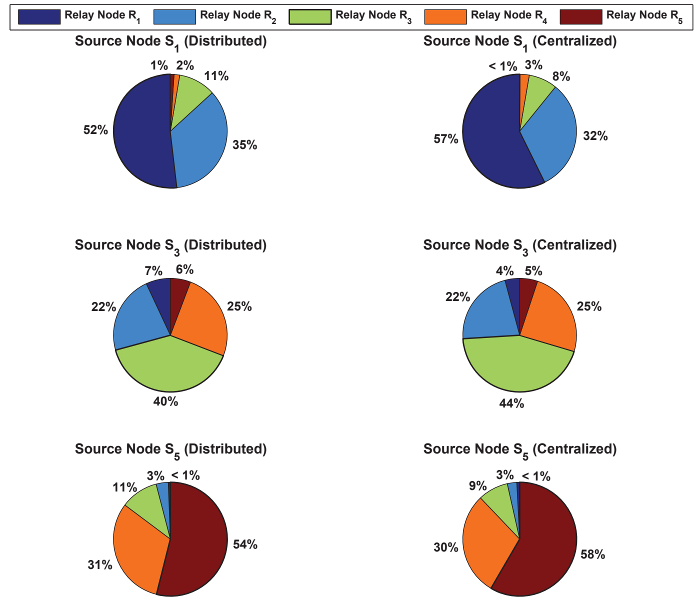

Fig. 3. Percentage of Relay Selection - Scenario 3 - ERP Allocation Scheme

To get a better idea of how the proposed distributed algorithm compares to the centralized one, Fig. 4 illustrates the achievable rate of each source node and the network sum-rate under the different scenarios. Under Scenarios 1 and 2, the distributed algorithm results in comparable rate for each source node under the ERP allocation scheme. Also, source S_1 benefits the most from the higher transmit power available at relay R_1, under the URP allocation scheme in all three scenarios. With respect to Scenario 3, it is observed that source S_3 achieves the highest rate under the distributed allocation with the ERP allocation scheme, which is due to its location being closer to relays R_2, R_3 and R_4. Hence, it benefits more spatial diversity gains than the other two source nodes. In general, it is noticed that the achievable rates under the distributed algorithm are rather comparable under the ERP allocation scheme; however, this is not necessarily the case under the centralized algorithm. As for the network sum-rate, the centralized algorithm outperforms the distributed algorithm under all scenarios irrespective of the relay power allocation scheme, as expected. It is also noticed that the network sum-rate under the URP allocation scheme is lower than the ERP allocation scheme, which implies that relay selection is more beneficial when all the relays have equal transmit powers.

Now, the following relay selection schemes are compared with the proposed distributed relay selection algorithm:

Closest Relay Selection (CRS):
Each source always selects the closest relay to it.

Best Harmonic Mean Selection (BHMS):
Each source node S_j for $j \in \mathcal{N}$ selects the relay node with the best harmonic mean value $\left(\text{i.e. } \arg\max_{k\in\mathcal{K}} \left(|h_{j,k}|^{-2} + |h_{k,d}|^{-2}\right)^{-1}\right)$ [1].

Best Worst Channel Selection (BWCS):
Each source node S_j for $j \in \mathcal{N}$ selects the relay node whose worse channel is the best (i.e. $\arg\max_{k\in\mathcal{K}} \min\{|h_{j,k}|, |h_{k,d}|\}$) [1].

Random Relay Selection (RRS):
Each source node randomly selects a relay node, without any respect to its transmit power or whether it had already been selected by another source node.

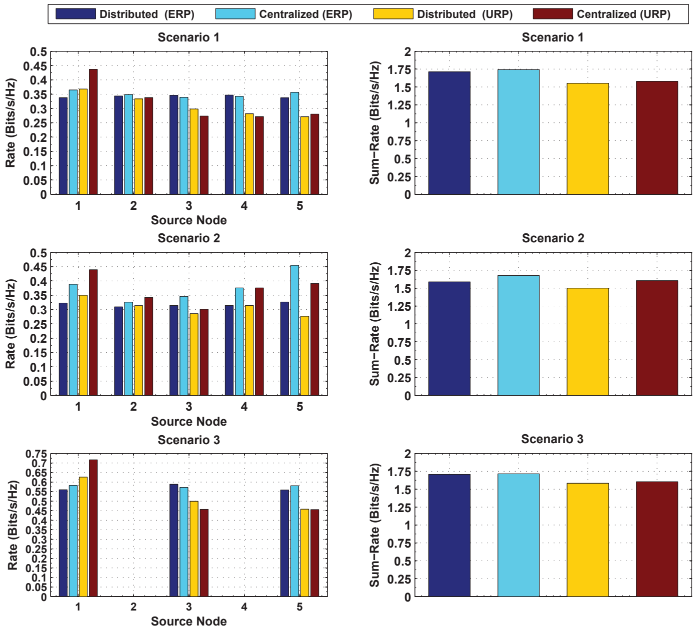

Fig. 4. Achievable Rate of Each Source Node and Network Sum-Rate Under the Distributed and Centralized Algorithms

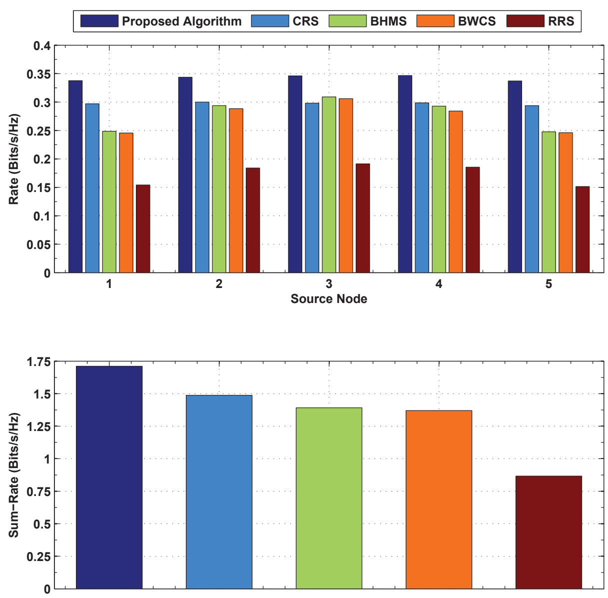

Fig. 5. Achievable Rate of Each Source Node and Network Sum-Rate Under Different Relay Selection Schemes vs. Distributed Relay Selection Algorithm - Scenario 1 - ERP Allocation Scheme

In Fig. 5, it can be seen that the achievable rates of each source node under the CRS, BWCS, BHMS, and RRS schemes are inferior to that of the proposed distributed relay selection algorithm, with the RRS scheme resulting in the lowest rate for each source node. This also can be seen for the network sum-rate. Lastly, one might argue that relay selection can be achieved with low-complexity through the CRS, BWCS, BHMS or RRS schemes. However, it is evident that they yield considerably lower network sum-rates than the proposed relay selection algorithm.

It should be noted that the distributed algorithm has extensively been verified to always converge to a Nash Equilibrium grouping, in a finite number of iterations. Specifically, under the ERP (URP) allocation scheme, the algorithm converges with an average number of iterations of 37 (39), 24 (25), and 18 (21) under Scenarios 1, 2 and 3, respectively. It was also determined that the minimum (maximum) number of iterations—irrespective of the relay power allocation scheme—is 25 (77), 15 (39), and 15 (31) under Scenarios 1, 2 and 3, respectively. Also, it is noted that increasing the number of source/relay nodes product (i.e. $N \cdot K$) increases the number of iterations until convergence. Moreover, by comparing scenarios 2 and 3 (where $N \cdot K = 15$ under both scenarios), it can be seen that Scenario 2 takes longer to converge than Scenario 3. This implies that the higher is the number of source nodes (for fixed $N \cdot K$), the longer is the convergence. Intuitively, this is due to the higher competition in relay selection, which results in more deviations until convergence.

VII. Conclusions

In this paper, a distributed relay selection algorithm has been proposed for relay selection in ad-hoc wireless networks. The proposed relay selection algorithm has been designed based on the Chinese restaurant game model to allow rational network source nodes to efficiently select relay nodes while taking into account negative network externality. It has been shown that the proposed algorithm converges to a Nash Equilibrium grouping. Several network scenarios and relay selection schemes have been evaluated and the proposed distributed algorithm has demonstrated its efficiency in relay selection. Finally, the proposed distributed algorithm has been shown to yield a network sum-rate that is comparable with that of centralized relay selection.

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